Whole geometry Finite-Difference modeling of the violin

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A Finite-Difference Modelling of the complete violin body is performed. A soft and a hard attack are modelled and the time series of the two cases are compared with recorded violin sounds. It shows up, that the violin top plate smoothes out the vibration applied by the violin bridge, because of it being kicked by the sawtooth tearing-off the bow from the string and nearly being left alone for the rest of the cycle and the inclosed air damping out very high frequencies in the violin top plate. The back plate is mostly driven by the inclosed air and adds a strong amplitude peak at the second half of the cycle. The ribs show very high frequencies and due to coupling to the top and back plate and also to the inclosed air, its vibration is very complex. The inclosed air adds a low sound to the system. The soft attack shows two phases in both, the modelled and the recorded sound, one before the steady Helmholtz sawtooth motion is established with slightly deviating eigenfrequencies of the whole system and the following quasi-steady part. The hard attack show the chaotic income much more prominent in both the modelled and recorded case. This scratchy sound at the tone beginning is caused by an interplay of large bowing pressure and velocity leading to changes in the change-to-slipping and change-to-sticking conditions of the violin bow on the string.

1 Introduction

The violin is largely discussed in the literature (see i.e. the basic analytical work of[3] or a summery of research papers in [6]). Here, problems of eigenfrequencies of the violin body, the bridge, the bow string interaction or the radiation of the violin are discussed. Also the transient phase of violin bowing has been investigated [5] [4] [8] [7].

2 Method

Here, the violin is presented as a Finite-Difference Model of the whole violin body. The different parts of the violin, the top plate, back plate, ribs, enclosed air, neck, bridge, sound post and the strings are modelled as a grid of node points according to their geometry. The model assumes the x-axes starting at the bottom of the violin with increasing values in the direction of the violin neck. The two sides of the violin are looked at as lying along the y-direction in positive (direction of sound post) and negative (direction of bass bar) parts. A software has been developed similar to an existing stand-alone Windows application of the classical guitar [1] which calculates, visualizes and allows manipulation of the material parameters or inputs, like knocking on the plates while the violin is moving on the screen in slow motion. By this visualization, the behaviour of the different plates can be looked at with much detail, i.e. the movement of the top plate in the area of the sound post making the top plate much more rigid or the coupling between bow, bridge and top plate.

The top and back plates are curved by suitable analytical functions, which divide the long rim of the plates along the x-axes at $y = 0$ into three sections. The starting and ending sections are curved in a way, that at the plate boundaries, the slope of the curvatures is zero, then the curvature increases to a maximum slope and decrease again up to a minimum slope point. The connection between these two minimum slope points on the violin is a steady slope line connecting these two points. Also the change of decrease in the y-direction according to the width of the violin in y-direction is treated with analytical functions. The description with functions was chosen, because it is assumed, that violin builders think of the plates being curved that way in an overall sense and also, because then only the two maximum slope points of the violin have to be chosen on the x-axes and in their height (z-direction) to obtain a reasonable curvature and the sharpening of the two slopes can be chosen as desired.

The top plate has additional mass because of the bass bar. The sound post is attached to the top plate (and the back plate) as one geometry. This has the advantage, that here no additional assumptions have to be made in terms of coupling of the plates to the sound post. The ribs are curved according to a classical violin geometry and additional masses are attached to it at the gluing areas of the top and back plate. The inclosed air is modelled, too. The f-holes are the radiating areas of the inclosed air and cut in the top plate. The neck is attached to the ribs with additional mass at the point of attachment and also simulates the finger board over the top plate. The four strings are also modelled as one dimensional Finite-Difference line of points. The driving mechanism of the bow is modelled as a changing sticking and slipping phase behaviour, where the bow pressure, the bowing velocity, and change-to-sticking as well as the change-to-slipping conditions are applied so that in a normal pressure bow-

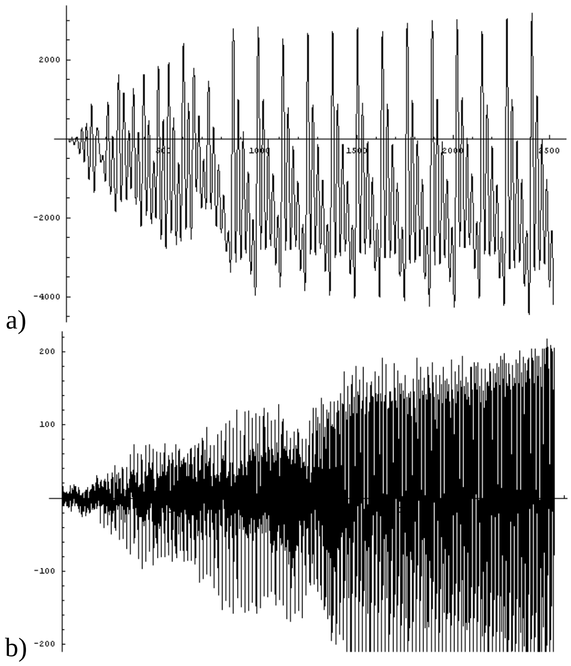


Figure 1: Comparison of the time series of a) 26 ms of the model and b) 200 ms of a recorded violin sound. The time series show the increase of amplitude and a break at the point, where bowing speed and bowing pressure reach the threshold of normal sawtooth motion. (As the recorded sound had this only after about 90 ms, the sound was chosen to be presented with 200 ms here.

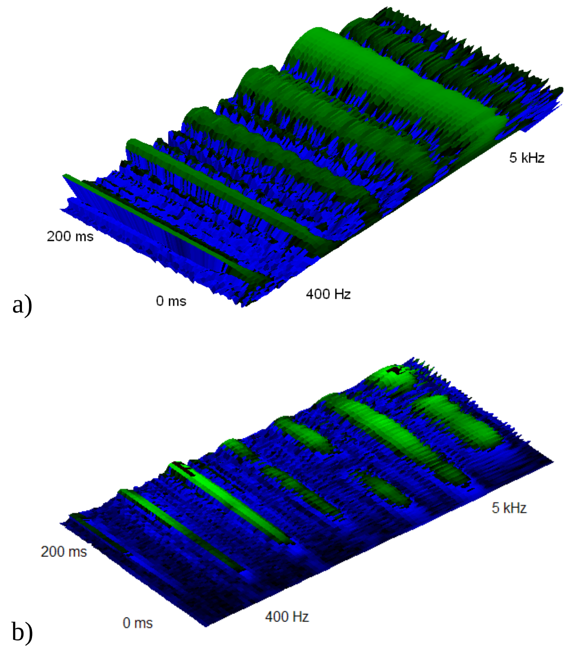


Figure 2: Comparison of the Wavelet Transform of a) the model and b) the recorded violin sound with soft attack both 200 ms. The overall behaviour is the same with increase of amplitude at the tone beginning and slight amplitude fluctuations in higher harmonics. The model shows a higher amplitude with the fundamental frequency which may be caused by the inclosed air being mixed to loud into the sound.

ing range the Helmholtz sawtooth motion occurs. Under a bowing pressure threshold the sawtooth motion breaks down into a rich bifurcation scenario and over the normal bowing pressure threshold, then the bowing pressure determines the fundamental pitch of the sawtooth, where in the normal bowing pressure region, the fundamental pitch is mostly determined by the string length to allow finger controlled pitch playing.

The differential equations used in the model are the fourth order equation of bending for the plates in their z-direction (the z-direction of the ribs are always outward like the ribs normal vector). Additional, the longitudinal or in-plane waves in the plate are modelled by a second order differential equation. Both are coupled by simple geometrical reasoning. Again, the model of the guitar was used here as a starting point, where the mathematical details were already tested. The coupling of the top and back plate to the ribs are a transformation of bending waves into longitudinal waves and vice versa, as the movement of the top plate in its z-direction is a movement of the ribs in their in-plane, or y-direction (and the other way round).

All parts of the violin are coupled to their neighboring parts. All couplings mentioned now also hold in the other direction. So the strings act on the bridge, the bridge on the top plate. The top plate acts on the ribs, the inclosed air and the sound post. The ribs and the inclosed air act on the back plate and finally the ribs act on the violin neck.

The radiation of sound from the different violin parts was modelled as an integration function over the parts, where a microphone position one meter in front of the parts was chosen. Here the differences in phase of the different locations of the parts to the microphone, as well as the different damping because of the changing distance to the microphone was taking into consideration. As no special room was applied around the violin, the sounds of the different violin parts can be added together with changing amplitude values for the different parts according to the listening position around the violin. Here, the top plate was looked at as the loudest part and so as a reference.

3 Results

The high e-string of the violin was played by the model and compared to a real violin sound. Two cases were looked at, a soft and a hard attack. As the initial transient of musical instruments is not only important for instrument identification, but also implies a lot of the ex-

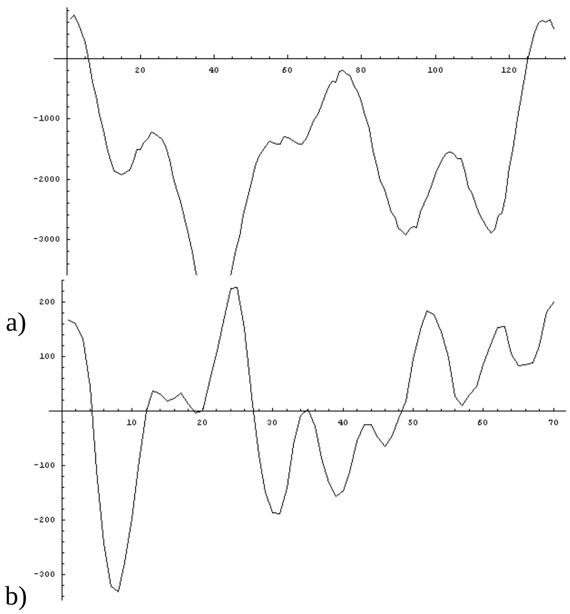


Figure 3: Comparison between one period cycle of a) the model and b) a recorded violin sound starting at the time point of tearing off the string from the bow. The overall structure is the same consisting of a large negative peak followed by a large positive again and then an fluctuating increase to the end of the cycle.

pression of the player, the transients of these two cases are compared. During the initial transient, the blowing pressure and velocity are changed. Both parameters are shown to be necessary to achieve realistic results of initial transients of violin sounds.

The normal bowing force F on the bowing area was assumed to be $1000N$ in the normal case. With the soft initial transient, within the first 20 ms, F was linearly increased from zero to 1000 and kept constant after 20 ms. With the hard attack case, the bowing pressure was taken to be 10 000 N at the beginning and linearly decreased to 1000 N again within 50 ms and kept constant beyond this point.

3.1 Soft attack

The soft attack was modelled by increasing the bowing pressure and the bowing speed linearly from zero to a value in the middle region of the normal sawtooth motion bowing pressure value within the first 20 ms. After this, the both parameters were held constant. In the figures below, the time series and wavelet-transforms of the resulting tone of pitch e^2 are shown.

Figure 1 shows the time series of a soft attack a) of the model and b) of a recorded violin sound. Both show the same behaviour of an increase of amplitude and some kind of a phase change in a) after $1/4$ of the sound and in b) at nearly half of the sound. Here, the normal bowing pressure and velocity is reached and the sawtooth motion

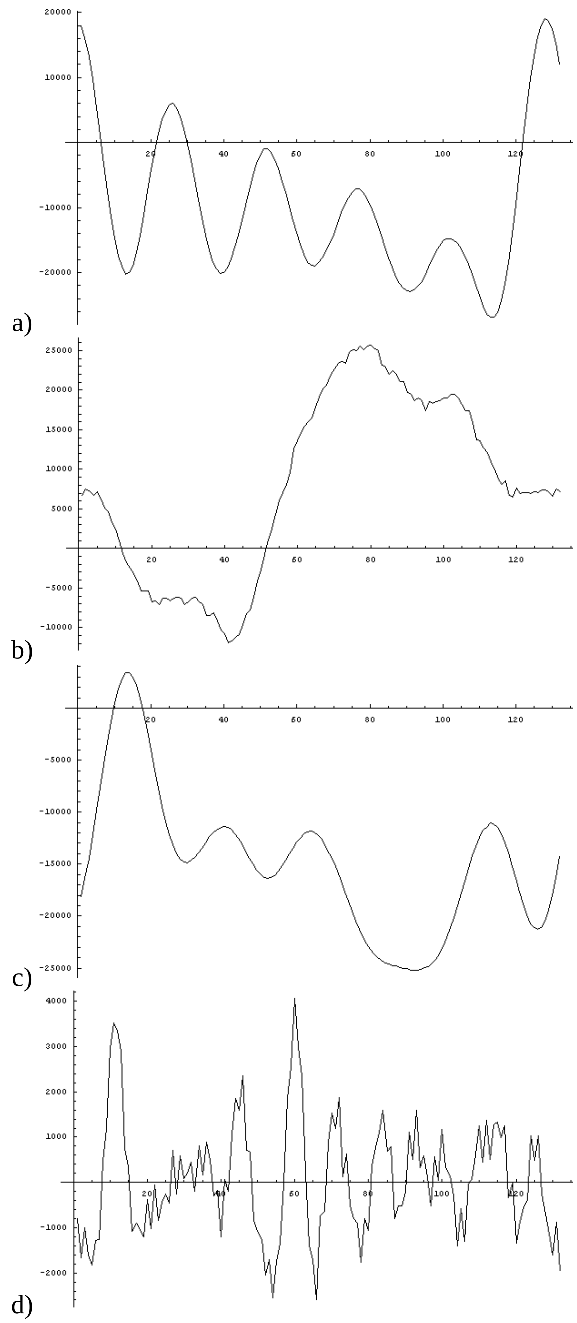


Figure 4: Time series of radiation of the different violin parts within one cycle of the previous figure of the model. a) Top plate, b) Back plate, c) Inclosed air, d) Ribs (see text for details).

of the quasi-steady state is reached. As the recorded violin sound took longer to reach this values compared to our model, the two different time intervals of presentation were chosen.

Figure 2 shows the Wavelet-Transforms of the two tones. They both show an increase of amplitude, but the soft attack of the real tone show a much larger change in amplitude even after 100 ms. Here, the bowing velocity and pressure have been changed further on which was not done in the model as this change is more or less arbitrary. Still, the model agrees with some periodic overtone amplitude fluctuations in the higher harmonics. Also, the model shows some slight deviations of the frequencies at the very beginning which is also present in the recorded wavelet-transform but is too low in amplitude here to be seen. But closer examination of the very beginning of the recorded sound show the same fluctuations which are caused by the bowing pressure being not large enough to establish even a kind of sawtooth motion. One could argue, that the chaotic region at the tone beginning is too large for a hard attack. Although this chaotic region is also there in the recorded sound - but with much less amplitude - it also may be appropriate to increase the bowing pressure even faster than the bowing speed.

Figure 3 shows the time series of the established sawtooth motion of a) the model and b) the recorded sound. It was taken from the time point of tearing-off the string from the bow. Both show a large negative response to the tearing-off followed by a large positive one and a overall but fluctuating increase of the time series up to the next tearing-off point. The recorded sound have these minimum and maximum amplitude values faster than the modelled sound. When listening to the two sounds, both clearly are violin timbres, but are not exactly the same, because the model was not constructed to fit the used violin but to show the overall behaviour of the instrument.

Figure 4 shows the time series of the different violin parts as integration over the geometry as mentioned above all in the same time interval as Figure 3 and so constructing the modelled time series of Figure 3. The overall movement of the time series is caused by the top plate. After the 'kick' of the bridge it starts vibrating on its own with decreasing amplitude and decreasing middle amplitude value, because the bridge still acts on the top plate with the slope of the sawtooth motion in the opposite direction as the tear-off kick. The inclosed air in Figure 4 c) show a time delayed response to the top plate as expected. Both, the top plate and the inclosed air interact in a way, that the top plate accelerates the air and the air calms down the higher harmonics of the top plate, because the wave velocity in air is much lower than in wood and the air load acts on the whole top plate. Without this kind of smoothing of the top plate vibration by the air, the top plate would much more vibrate like the sawtooth motion resulting in a much sharper sound. The inclosed air then adds a a very low sound to the violin tone. The ribs, which radiate much lower than the top plate show a very complex vibration. As the top plate acts on the ribs in the in-plane direction of the ribs, but this in-plane movement is transformed into a radiating bending movement within the plates, this bending movement has to interact with the inclosed air applying a force in the bending direction of the ribs. Still the eigenfrequencies of the ribs differ a lot

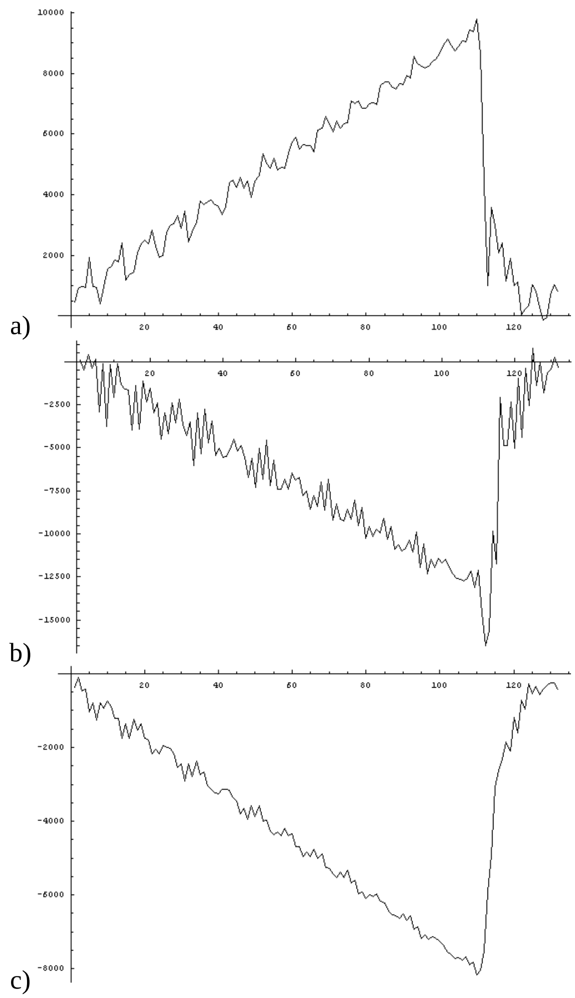


Figure 5: Time series of the string and bridge in the temporal range of the previous two figures: a) string point at bridge on string, b) in-plane motion of the bridge integrated over its area, c) motion of the bridge food near the sound post. (see text for details)

from those of the top plate or the inclosed air, being much higher in an upper middle frequency region. These three vibrations cause the ribs to add a high frequency sound to the violin tone. Finally the back plate is a combination of the inclosed air with a little bit of rib interaction. As the inclosed air changes its vibration during the travel through the violin ribs height, it acts on the back plate more in an overall fashion of high and low amplitude values. The high amplitude value of the second half of the back plate vibration during the shown cycle is the cause for the time series in Figure 3 a) to rise to the end again. Adding more back plate sound to the time series in Figure 3 a) (like putting the ear near the back plate of the violin) results in a larger increase at the second half of the period.

Figure 5 shows the time series of a) the string at the bridge point, b) the bridge in its in-plane vibration integrated over the whole bridge and c) of the bridge foot near the sound post. Clearly, the sawtooth motion can be seen with slight fluctuations caused by the string being split in two when gluing to the bow and the small part between bow and bridge vibrating with the fundamental frequency of that string length during sticking. The bridge itself adds a little bit of vibration to the sawtooth motion, but this is not much as expected. The fundamental frequency of the in-plane motion of the bridge is very high and so the bridge more or less acts like a very stiff geometry. Additionally, as the bridge has the possibility to move within itself because of its geometry of two feet, this damps out the small fluctuations a bit and results in a more rigid sawtooth motion as the string supplies.

3.2 Hard attack

The hard attack was modelled with changing bowing pressure values but here also with changing velocity values. The hard bowing pressure at the beginning was accompanied by a linearly increasing bowing velocity. As the velocity was high at the beginning, no sawtooth motion could occur and many small impulses were travelling along the string quite randomly. As the velocity got higher, some of the impulses were cancelled out because the bow was displacing the string now with larger amplitudes so the back coming waves were no longer able to tear off the bow from the string any more. But each successful tearing-off in this region lead to a much larger impulse travelling along the string than before. So the hole interaction then takes place one step higher. This leads to a scratchy sound, because no periodicity is reached. But as the impulses still travel along the string, get reflected at the string end and then having a change to tear off the string from the bow again after one cycle of travelling, a kind of periodicity is still established. This is the reason, why in the Wavelet-Transformations in Figure 7 something like a pitch seems to be established. The scratchy

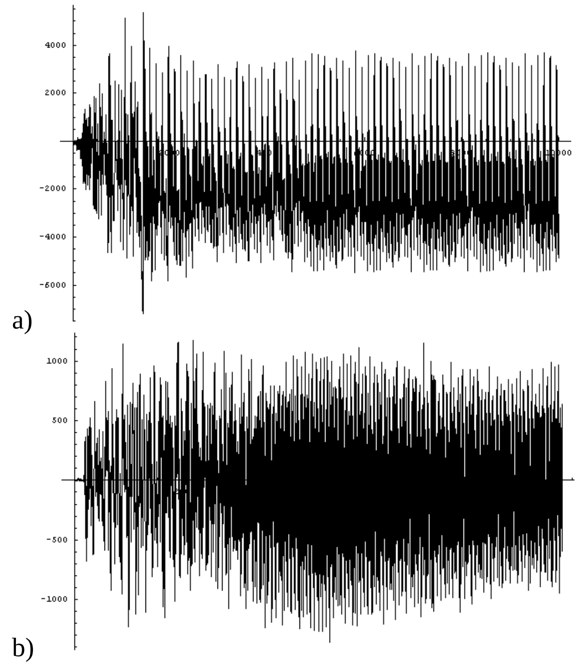
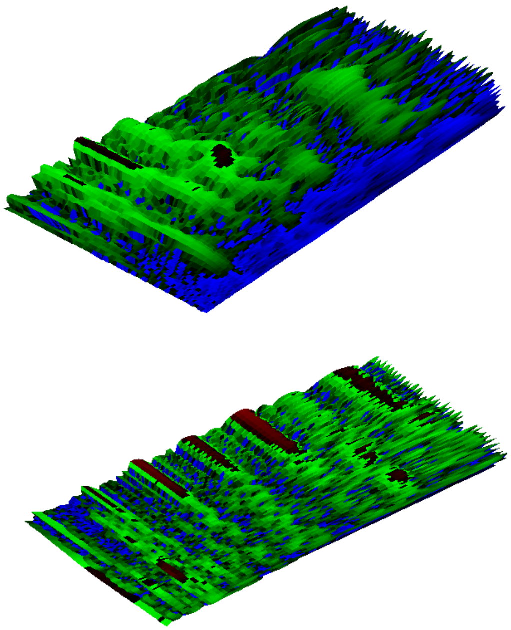


Figure 6: Comparison between the time series of a hard and scratchy attack between a) the model and b) a recorded violin tone. The model sound is 100ms, the recorded is 200ms. Again, the recorded sound has a longer attack, which does not change the overall behaviour. Again like with the soft attack the time series relaxes after the bowing pressure and velocity have reached a normal bowing region.

tone then comes from all the small impulses travelling reaching the bridge and being radiated by the instrument body.

The modelling of the hard attack was not possible with too high bowing pressure. If the normal pressure regime was left to the higher end, the string had no chance to tear off the bow from the string again. On the other side, if the velocity was too high at the beginning, the string was displaced quite large at the beginning and also here, the back coming impulses had no change to tear off the string from the bow. Only if the bowing pressure is rises slowly with the velocity too, a soft attack could be established. But again, to avoid a noisy attack, the bowing pressure has to be raised much more quickly to its sawtooth regime value. An attack with velocity and pressure fixed over all length of the initial transient, an astonishing quick establishment of the normal sawtooth motion was possible. Listening to sounds like that is a mixture between a soft and a hard attack. No scratchy sound can be heard, but as the amplitude rise time is very short the sound is perceived more like a hard income than like a soft one.

a)



b)

Figure 7: Comparison between the Wavelet-Transforms of the time series shown in the previous figure. Both show a struggling of the string eigenfrequencies within the initial transient up to the point where normal bowing pressure and velocity values are reached.

4 Conclusion

The initial transient of violin tones can be modelled realistically by varying only the bowing pressure and the bowing velocity, where the bowing velocity plays the much more important role. Changes in the bowing pressure over or under the normal sawtooth regime values lead to unrealistic attacks. The soft attack is reached by increasing the bowing pressure very fast to a normal value while the bowing speed should be increased slowly. A normal sawtooth motion is reached here very fast but with low amplitude. The hard attack need a low bowing speed at the beginning and a normal bowing pressure value so that the travelling impulses on the string have the change to tear off the string from the bow quite randomly. Still as the impulses have to travel all the way along the string, a quasi periodic motion is established at the tone beginning. The third possibility of attack is a constant value of velocity and bowing pressure right from the start. This leads to an attack being in a middle region between soft and hard attack, because the amplitude is there immediately, but no real scratchy region occurs during the initial transient phase.

The violin body behaviour can be viewed as a combination between top plate, back plate, ribs and inclosed air. The top plate smoothes out the hard sawtooth motion of the bridge mostly by the damping of the air inclosed. The air motion is similar to the top plate motion but lacking the clear modulation and amplitude decrease during one cycle. The back plate show just one large amplitude change during one period with lowering the overall amplitude of the radiated sound in the first half of one cycle and increasing in in the second half. The ribs show a very complicated radiation pattern, being influenced by the top plate, the back plate and the ribs and itself having a high resonance spectrum.

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